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FISH 558

Lab 2 HW

1/29/2024

1. **Continuous logistic growth**
   1. **Plot of abundance for 140 years under two scenarios**

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Figure 1: Plot of abundance (number of individuals) vs time (years) in two logistic growth scenarios starting at 10% of carrying capacity. The pink line represents a scenario where intrinsic rate of increase r = 0.05/year, and the blue line represents a scenario where it is 0.2/year.

* 1. **What is the population size after 10 years?**

**0.05 scenario**: N.cont.1[11] = **15,483** individuals

**0.2 scenario**: N.cont.2[11] = **45,085** individuals

*Called N[11] because N[1] represents year 0, not year 1*

* 1. **What percentage of the carrying capacity does that equate to?**

**0.05 scenario**: (15,483 / 100,000) \* 100 **= 15.48**%

**0.2 scenario**: (45,085 / 100,000) \* 100 **= 45.09**%

* 1. **How many years to reach 75% of K?**

Solve logistic growth equation for t:

**A close-up of a math equation

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Figure : Scan of steps needed to solve the logistic growth equation for t, in order to find t when N = 75% of K.

Calculate t using Nt = K \* 0.75

**0.05 scenario**: (15,483 / 100,000) \* 100 **= 65.91674** years

**0.2 scenario**: (45,085 / 100,000) \* 100 **= 16.47918** years

(see code chunk labeled 1d at end of doc for the code used for the calculation)

* 1. **What is the max growth rate for each scenario?**

Solve using equation , where N = K \* 0.5.

**0.05 scenario**: =  **1,250** individuals/year

**0.2 scenario**: =  **5,000** individuals/year

1. **Discrete-time patterns**
   1. **Generate plots**

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Figure 3: Plot of abundance (number of individuals) vs time (years) in two discrete-time logistic growth scenarios. The pink line represents a scenario where intrinsic rate of increase r = 2.0/year, and the blue line represents a scenario where it is 3.0/year.

* 1. **Why do these patterns occur here but not in continuous-time model?**

Since the discrete-time model calculates values at distinct points in time, it gains the appearance of “jumping” from point to point. Continuous-time models don’t display this because they calculate instantaneous values throughout time, so their appearance will be smoother. The patterns are particularly “spiky” and pronounced because the rates of increase are so high. The blue line, for example, has an intrinsic rate of increase of 3.0, meaning it has the potential to triple its population at any year where the population is small and the dampening effect of being close to carrying capacity is negligible.

1. **Comparison of growth models and logistic models with population size, growth rate, and per capita growth rate**
   1. **Generate plots**

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Figure 4a: Plot of abundance (number of individuals) vs time (years) for two growth scenarios. The pink line represents an exponential growth model, while the blue line represents a logistic growth model with a carrying capacity of 100. Both models start at N=2 at t=0, and both have an intrinsic rate of increase 0.1.

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Figure 4b: Plot of production (dN/dt) vs. abundance (number of individuals) for two growth models with similar parameters. The pink line corresponds to an exponential model, while the blue curve corresponds to a logistic model.

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Figure 4c: Plot of per capita growth rate ((dN/dt)/N, production per individual) vs. abundance (number of individuals) for two growth models with an intrinsic growth rate r = 0.1/year. The pink line corresponds to an exponential model, while the blue curve corresponds to a logistic model.

* 1. **How do the exponential and logistic models compare?**

**Figure 4a, abundance vs. time**: the logistic model levels off at carrying capacity K at around t = 75, after which the population stops growing. By contrast, the exponential model has the population continue to skyrocket after this, growing so much larger that the scale of the Y-axis had to be limited to be able to see the details of the logistic curve. By around t = 25, the curves had already begun to noticeably separate.

**Figure 4b, production vs. abundance**: the logistic model shows a parabolic curve, where production starts off low and increases to its maximum at N = 50, which is half of K. The exponential model shows a very different pattern, as it has a simple linear increasing relationship, where production always grows and never slows down.

**Figure 4c, per capita growth rate vs. abundance**: the logistic model shows a negative linear relationship, whereby as the population grows, per capita growth rate is always decreasing. This continues until carrying capacity N = K, at which point the per capita growth rate hits 0 and the population stops growing. The exponential model never stops growing and always maintains the same per capita growth rate, resulting in a flat line relationship.

* 1. **How does population plot relate to the growth rate plot? Where is growth rate largest?**

Figures 4a and 4b are related because the y-axis on 4b (production) is the slope of the curves in 4a. This can be seen clearly on the blue logistic curves, as 4b hits 0 production at N = 0 and N = 100, where slope = 0 on 4a. Since the exponential curve has constantly increasing slope not constrained by a carrying capacity, its relationship is linear.

Already answered in question 4b, but the maximum production (growth rate) is found at K = 50, which is the top of the parabolic curve in figure 4b and corresponds to where the sigmoidal curve in 4a had the highest slope.

* 1. **How does the growth rate plot relate to per capita growth rate plot?**

Figures 4b and 4c are related because the y-axis on 4c (production) is derived from the y-axis on figure 4b divided by N, being (dN/dt)/N. This shows us that the per-capita growth rate of the logistic curve is constantly decreasing, and that the per capita growth rate is always one tenth of the population size, indicated by a flat line at 0.1, which corresponds with the intrinsic rate of increase provided, r = 0.1.

1. **Additional questions**
   1. **How many hours?**

3.5 hours total

* 1. **Group work**

(I am in the top left background, blurry, and wearing green)

A group of people in an office

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1. **Stochastic continuous logistic growth model**

**a & b. Generate a plot with r = 0.05, and one with a mean of 0.2**

I elected to address both these questions together by including both simulations on the same plot.

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Figure 5: Plot of abundance (number of individuals) vs time (years) in two continuous-time logistic growth scenarios. The pink line represents a scenario where the mean intrinsic rate of increase is 0.05/year, and the blue line represents a scenario where it is 0.2/year. Stochastic population effects have been added to both models, with r values being normally distributed with a standard deviation of 0.01.

1. **Describe differences between 5a and 5b**

The most noticeably differences in the two scnearios in figure a and b concern the speed at which they reach carrying capacity and the general “spikiness” caused by the added stochasticity. The blue line corresponds to a higher intrinsic rate of increase, which helps explain why it hits carrying capacity earlier. The pink line, with a lower intrinsic rate of increase, does not reach carrying capacity within the time window of t = 0 to 120. The blue line shows that reaching carrying capacity has the effect of stabilizing the population and dampening the oscillations that cause the extra “spikiness” seen on the pink line, which increases as it grows.

**Appendix: CODE pasted below**

library(ggplot2)

library(tidyverse)

#define parameters

K = 100000

r1 = 0.05

r2 = 0.2

t=c(0:140)

N0=10000

**# question 1 ####**

**#1a: generate plot**

N.cont.1 = K/(1+(((K-N0)/N0)\*exp(-r1\*t)))

N.cont.2 = K/(1+(((K-N0)/N0)\*exp(-r2\*t)))

df.cont = data.frame (t = t, N1 = N.cont.1, N2 = N.cont.2)

plot1 <- ggplot(data=df.cont, aes(x=t, y=N1, color = "r = 0.05"))+geom\_line()

plot1 + geom\_line(aes(x=t, y=N2, color = "r = 0.2")) + theme\_bw() +ylab("N")

**#1b: pop size after 10 years**

N.1.10 <- N.cont.1[11]

N.2.10<-N.cont.2[11]

**#1c: percent K**

percent.1 <- (N.1.10/K) \*100

percent.2 <- (N.2.10/K) \*100

**#1d: time to reach 75% of K**

Ntarget = K \* 0.75

#defining numerator and denominator separately to reduce parentheses confusion

numerator = (K/Ntarget)-1

denominator = (K-N0)/N0

t.target.1 = log(numerator/denominator)/-r1

t.target.2 = log(numerator/denominator)/-r2

**#1e: growth rate (dN/dt) at K/2 (max)**

N.max.growth = K\*0.5

dN.dt.max.1 = r1\*N.max.growth\*(1-(N.max.growth)/K)

dN.dt.max.2 = r2\*N.max.growth\*(1-(N.max.growth)/K)

**# question 2 ####**

r3=3

r4=2

K=1000

N0=100

t=c(0:50)

#generate first data set

N.logist.discr.1 = rep(NA, length=length(t))

N.logist.discr.1[1] = N0

for(i in 1:(length(t)-1)) {

N.logist.discr.1[i+1] = N.logist.discr.1[i] + r3\*N.logist.discr.1[i] \*(1-N.logist.discr.1[i]/K)

}

#generate second data set

N.logist.discr.2 = rep(NA, length=length(t))

N.logist.discr.2[1] = N0

for(i in 1:(length(t)-1)) {

N.logist.discr.2[i+1] = N.logist.discr.2[i] + r4\*N.logist.discr.2[i] \*(1-N.logist.discr.2[i]/K)

}

**#2a: plots**

df.discr<- data.frame(t=t, N1 = N.logist.discr.1, N2=N.logist.discr.2)

plot2 <- ggplot(data=df.discr, aes(x=t, y=N1, color = "r = 3.0"))+geom\_line()

plot2 + geom\_line(aes(x=t, y=N2, color = "r = 2.0")) + theme\_bw() +ylab("N")

**# question 3 ####**

N0=2

r=0.1

K=100

t=c(0:100)

**#3a: plots**

#plot N vs t

N.exp = N0\*exp(r\*t)

N.log = K/(1+(((K-N0)/N0)\*exp(-r\*t)))

df.3 = data.frame (t=t, N.exp=N.exp, N.log=N.log)

plot3 <- ggplot(data=df.3, aes(x=t, y=N.exp, color = "Exponential"))+geom\_line()

plot3 + geom\_line(aes(x=t, y=N.log, color = "Logistic")) + theme\_bw() +ylab("N") +ylim(c(0, 500))

#plot dN vs N

dN.dt.log = r\*N.log\*(1-(N.log)/K)

dN.dt.exp = r\*N.exp

df.4 <- cbind(df.3, dN.dt.exp, dN.dt.log)

plot4 <- ggplot(data=df.4, aes(x=N.exp, y=dN.dt.exp, color = "Exponential"))+geom\_line()

plot4 + geom\_line(aes(x=N.log, y=dN.dt.log, color = "Logistic")) + theme\_bw() +

ylab("dN.dt") +xlab("N")+ylim(c(0,5))+xlim(c(0,110))

#plot dN/N vs N

df.4<- mutate(df.4, dN.dt.exp/N.exp)

df.4<- mutate(df.4, dN.dt.log/N.log)

plot5<- ggplot(data=df.4, aes(x=N.exp, y=dN.dt.exp/N.exp, color="Exponential"))+

geom\_line()+geom\_line(aes(x=N.log, y=dN.dt.log/N.log, color="Logistic"))+

theme\_bw()+ylab("dN.dt/N") +xlab("N")+xlim(c(0,110))

**# question 5 ####**

**#5a/5b: plots**

K = 1000

sigmar = 0.01

N0=20

t=c(0:120)

r.values= rnorm(t, mean=0.05, sd = sigmar)

N.cont.5 = K/(1+(((K-N0)/N0)\*exp(-r.values\*t)))

r.values.2= rnorm(t, mean=0.2, sd = sigmar)

N.cont.6 = K/(1+(((K-N0)/N0)\*exp(-r.values.2\*t)))

df.cont.5 = data.frame (t = t, N5 = N.cont.5, N6 = N.cont.6)

plot1 <- ggplot(data=df.cont.5, aes(x=t, y=N5, color = "rbar = 0.05"))+geom\_line()

plot1 + geom\_line(aes(x=t, y=N6, color = "rbar = 0.2")) + theme\_bw() +ylab("N")